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13. ABSTRACT (Maximum 200 words)

Together with students and postdocs, the PI has worked on the mathematical aspects and applications of various tools in time-frequency or time-scale analysis. they have brought a deeper understanding to the geometry of redundant representations (frames), and shown the usefulness of frames for multiple description transmission, designed to withstand partial loss of information if some of the channels fail. They have also worked on subdivision schemes for non-uniform data, which can be used to provide efficient data compression for irregularly spaced data which are a superposition of fairly smooth signals with transients. Finally, they have obtained results on several reallocation schemes, aimed at extracting different components from complex signals, without too much "pollution" from the analyzing tool.

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FINAL REPORT

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Tools for Time-Frequency Analysis

Frames of windowed Fourier transforms and wavelets: work in collaboration with Radu Balan, a graduate student supported by this grant.

Frames provide redundant descriptions of vectors in a Hilbert space; that is, the sequences $(<f, \varphi_k>)_{k \in K}$, where the $(\varphi_k)_{k \in K}$ constitute a frame in $\mathcal{H}$ and f roams over all of $\mathcal{H}$, do not fill all of $\ell^2(K)$ because of linear dependencies between the φ_k reflected by the inner products $<f, \varphi_k>$. The range of the frame, i.e. the subspace of $\ell^2(K)$ given by $\{(<f, \varphi_k>)_{k \in K}; f \in \mathcal{H}\}$, characterizes the frame up to an isomorphism: if two frames $(\varphi_k)_{k \in K}$ and $(\psi_k)_{k \in K}$ have the same range, then there exists a banded operator A with banded inverse such that $\psi_k = A\varphi_k$. This and more detailed observations led to [1], a paper studying the geometric structure of frames.

On the other hand, if the ranges of two frames are orthogonal subspaces of $\ell^2(K)$, then we call the frames themselves orthogonal. In this case it is possible to use them for multiplying. This means that if f, g are two (unrelated) functions in $\mathcal{H}$, we can recover both f and g from the <u>single</u> sequence $(\alpha_k)_{k \in K}$, with $\alpha_k = <f, \varphi_k> + <g, \psi_k>$. This construction can be extended to the case where the ranges of the two frames make a non-zero angle [2].

In a dual approach, we are also investigating how frames can be used for multiple descriptions. In this case, a redundant description of a signal gets split into different channels, each of which carries only partial information. The idea is then to design the scheme optionally so that even if some of the channels are interrupted, the reconstruction from the remarking partial information is best possible. This is work in progress, with Vinay Vaishampayan.

In addition, Radu Balan also proved several stability results. For instance, he proved that for any wavelet Riesz basis, one can stretch or shrink the translation parameter slightly and still retain a Riesz basis [3].

Time-frequency localization:
The squeezed wavelet transform used in the joint work with S. Maes [4] is a special case of a reassignment method. With Patrick Flandrin and Eric Chassande-Mottin, the PI has studied other reallocation schemes. Here one starts from a time-frequency decomposition, in which one tries to "undo" the blurring effect of the time-frequency tool so as to obtain more sharply focused time-frequency representations. We have succeeded in finding some forms that reallocate in both the time and frequency variables, while still giving exact reconstruction.

Moreover, we have studied a "differential reassignment scheme" that lets a signal-dependent vector flow act on the Wigner transform; under this flow, the transform concentrates into attracting basins, which makes it possible to use this as a tool to decompose signals into different components [5]. In a different approach, we are exploring, with Fabrice Planchon, the mathematical aspects of smoothing the Wigner-Ville distribution with Gaussians that are locally oriented and stretched in a signal-adaptive way, given by geometric properties of the Wigner distribution itself. This results in a positive time-frequency representation without the interference terms that characterize Wigner distributions, but that is nevertheless sharper than most smoothed Wigner-Ville distributions.

Lossless compression:
In some applications, the data that one wants to compress are already in the form of integers (digitized grey scale images, for instance). With A.R. Calderbank, W. Sweldens, and B.L. Yeo, the PI developed schemes that adapt wavelet transforms into schemes that map integers to integers. These turn out to be generalizations of schemes that exist in the literature. We can use our method starting from any wavelet

filter bank, and for some of these filter banks we obtain better results for subband-based lossless image compression than previously known [6].

Recently, B.L. Yeo managed to push these results even further, and our approach now leads to better results than any other lossless scheme, whether or not it uses subband filtering [7].

Smoothness for unequally spaced subdivision:
With Igor Guskov (Ph.D. student partially supported by this grant) and Wim Sweldens (Bell Laboratories) we studied the smoothness for subdivision schemes with non-equally spaced points. Subdivision schemes underlie every construction of wavelet bases linked with fast algorithms. The smoothness generated by the subdivision scheme is the key to the smoothness of the wavelets as well as to their potential for data compression, or for the sparse yet accurate representation of multiscale phenomena. Although we have a very thorough and detailed understanding of these smoothness issues in the case where the wavelet centers are placed on regularly spaced lattices, very little is know for the case where the wavelet centers are irregularly spaced. A better understanding of these issues is crucial for many applications, including compression, visualization or editing of signals or surfaces given by irregularly sampled data. We have shown that although neither Fourier nor spectral methods apply in the irregularly spaced case, we can nevertheless still use "commutation formulas," which are constraints on the subdivision filters, to obtain smooth functions [8].

References:
[1] R. Balan, **Equivalence Relations and Distances between Hilbert Frames**, submitted to Proceedings of the AMS

[2] R. Balan, **Extensions of No-Go Theorems to Many Signal Systems**, to appear in the *Proceedings of the Special Sessions on Wavelets, Multi-Wavelets and Applications*, AMS-MAA Meeting, San Diego 1997

[3] R. Balan, **Stability Theorems for Fourier Frames and Wavelet Riesz Bases**, *Journal of Fourier Analysis and Applications*, (3) no. 5, (1997), 499-504

[4] I. Daubechies and S. Maes, **A Nonlinear Squeezing of the Continuous Wavelet Transform Based on Auditory Nerve Models**, in *Wavelets in Medicine and Biology*, pp. 527-546, eds., A. Aldroubi and M. Unser, CRC Press, 1996

[5] F. Auger, E. Chassande-Mottin, I. Daubechies and P. Flandrin, **Differential Reassignment**, submitted to *IEEE Sig. Proc. Letters*

[6] A.R. Calderbank, I. Daubechies, W. Sweldens, and B.L. Yeo, **Wavelet Transforms that Map Integers to Integers**, to appear in *Applied and Computational Harmonics Analysis*

[7] N. Memon, X. Wu and B.L. Yeo, **Entropy Coding Techniques for Lossless Image Compression with Reversible Integer Wavelet Transforms**, IBM Research Technical Report RC21010, Oct 1997, *IBM CyberJournal*, submitted to International Conference on Acoustics, Speech and Signal Processing, 1998

[8] I. Daubechies, I. Guskov, and W. Sweldens, **Regularity of Irregular Subdivision**, submitted to *Constructive Approximation*

EXECUTIVE SUMMARY

Personnel Supported:

Ingrid Daubechies: Professor, Program in Applied and Computational Mathematics, one month in each of the summers 1995, 1996, 1997.

Radu Balan: graduate student in the Program in Applied and Computational Mathematics, stipend in 1995-96 (tuition paid by University fellowship), summer 1996, stipend and tuition 1996-97, summer 1997, stipend and tuition in 1997-98.

Igor Guskov: graduate student in the Program in Applied and Computational Mathematics, summer 1997

Jonathan Mattingly: graduate student in the Program in Applied and Computational Mathematics, one-half support during spring 1996.

Fabrice Planchon: graduate student in the Program in Applied and Computational Mathematics, partial postdoctoral support, September 1997.

Publications:
Apart from [1], [2], [3], [4], [5], [6], [7], [8] above,

A. Cohen, I. Daubechies, and G. Plonka, **Regularity of Refinable Function Vectors**, submitted to *Journal of Fourier Analysis and Applications*

A. Cohen, I. Daubechies, and A. Ron, **How Smooth is the Smoothest Function in a Given Refinable Space?** submitted to *Applied and Computational Harmonic Analysis*

R. Balan, **An Uncertainty Inequality for Wavelet Sets**, to appear as a LETTER TO THE EDITOR in *Applied and Computational Harmonic Analysis*

I. Daubechies and A. Gilbert, *Harmonic Analysis, Wavelets, and Applications*, lecture series presented at the Institute for Advanced Study's Park City Mathematics Institute, to appear in the Graduate Summer School Lectures, American Mathematical Society, July 1995

I. Daubechies, **Where Do Wavelets Come From? — A Personal Point of View**, *Proceedings of the IEEE Special Issue on Wavelets* 84 (no. 4), pp. 510-513, April 1996

M. Unser and I. Daubechies, **On the Approximation Power of Convolution-based Least Squares Versus Interpolation**, submitted to *IEEE Trans. Sig. Proc.*, January 1996

Interactions:
Together with Stuart Schwartz (Electrical Engineering) and René Carmona (Civil Engineering), and now also joined by Michael Orchard (Electrical Engineering), the PI organized a weekly brown bag seminar on time-frequency and time-scale methods, featuring inside as well as outside speakers. Participants are faculty, postdocs, and graduate students from Electrical Engineering, Civil Engineering, Mathematics, Applied and Computational Mathematics, and Computer Science. These meetings were very lively, and usually extended well into the afternoon with interesting discussions. These often led to the formulation of new problems to look at; they also gave rise to new collaborations. We are also considering organizing one or two one-day meetings of the same nature with a wider circle of time frequency researchers in the Northeast.

Honors and Awards:
American Mathematical Society Ruth Lyttle Satter Prize, 1997.

American Mathematical Society Steele Prize, 1994.

Lifetime achievement honors:

    MacArthur Fellowship for 1992-97.

    Member of American Academy of Sciences since 1993.